

Fitting the Union2.1 SN Sample with the $R_h = ct$ Universe

F. Melia¹

Department of Physics, The Applied Math Program, and Steward Observatory,

The University of Arizona, AZ 85721

`fmelia@email.arizona.edu`

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ABSTRACT

The analysis of Type Ia supernova data over the past decade has been a notable success story in cosmology. These standard candles offer us an unparalleled opportunity of studying the cosmological expansion out to a redshift of ~ 1.5 . The consensus today appears to be that Λ CDM offers the best explanation for the luminosity-distance relationship seen in these events. However, a significant incompatibility is now emerging between the standard model and other equally important observations, such as those of the cosmic microwave background. Λ CDM does not provide an accurate representation of the cosmological expansion at high redshifts ($z \gg 2$). It is therefore essential to re-analyze the Type Ia supernova data in light of the cosmology (the $R_h = ct$ Universe) that best represents the Universe’s dynamical evolution at early times. In this paper, we directly compare the distance-relationship in Λ CDM with that predicted by $R_h = ct$, and each with the Union2.1 sample, and show that the two theories produce virtually indistinguishable profiles, though the fit with $R_h = ct$ has not yet been optimized. This is because the data cannot be determined independently of the assumed cosmology—the supernova luminosities must be evaluated by optimizing 4 parameters simultaneously with those in the adopted model. This renders the data compliant to the underlying theory, so the model-dependent data reduction should not be ignored in any comparative analysis between competing cosmologies. In this paper, we use $R_h = ct$ to fit the data reduced with Λ CDM, and though quite promising, the match is not perfect. An even better fit would result with an optimization of the data using $R_h = ct$ from the beginning.

Subject headings: cosmic microwave background – cosmological parameters – cosmology: observations – theory – distance scale – stars: supernovae

1. Introduction

We now have several methods at our disposal for probing the expansion history of the Universe, but none better than the use of Type Ia supernovae (Perlmutter et al. 1998, 1999; Garnavich et al. 1998; Schmidt et al. 1998; Riess et al. 1998). Producing a relatively well-known luminosity, these events function as reasonable standard candles, under the assumption that the power of both near and distant events can be standardized with the same luminosity versus color and light-curve shape relationships.

Over the past decade, the combined efforts of several groups have led to an impressive accumulation of events and an ever improving quality of individual measurements. More recently, Kowalski et al. (2008) devised a framework for analyzing such data sets in a homogeneous manner, creating an evolving compilation currently known as the Union2.1 sample (see Suzuki et al. 2012 for its most recent incarnation), which contains 580 supernova detections. At the highest redshifts ($z > 1$), the Hubble Space Telescope plays a key role, not only in discovering new events, but also in providing high-precision optical and near-IR lightcurves (see, e.g., Riess et al. 2004; Kuznetsova et al. 2008; Dawson et al. 2009). While at lower redshifts, extensive surveys from the ground continue to grow the sample at a sustainably high rate (see references cited in Suzuki et al. 2012).

These events constrain cosmological parameters through a comparison of their apparent luminosities with those predicted by models over a range of redshifts. The key point of this exercise is that models with different expansion histories predict specific (and presumably distinguishable) distance vs. redshift relationships, which one may match to the observations for a comparative analysis. The consensus today appears to be that Λ CDM offers the best explanation for the redshift-luminosity distribution seen in these events, and

¹John Woodruff Simpson Fellow.

observational work is now focused primarily on refining the fits to improve the precision with which the model parameters are determined.

When these efforts are viewed through the prism of a more comprehensive cosmological study, however, the situation is not quite so simple. The reason is that although Λ CDM does very well in accounting for the properties of Type Ia supernovae, it does poorly in attempts to explain several other equally important observations, particularly the angular distribution of anisotropies in the cosmic microwave background (CMB) and the clustering of matter. For example, the angular correlation function of the CMB is so different from what Λ CDM predicts that the probability of it providing the correct expansion history of the Universe near recombination is less than $\sim 0.03\%$ (Copi et al. 2009; Melia 2012b). And insofar as the matter distribution is concerned, Λ CDM predicts a distribution profile that changes with scale, seemingly at odds with the observed near universal power-law seen everywhere below the BAO scale (~ 100 Mpc), leading Watson et al. (2011) to categorize it as a “cosmic coincidence.”

This disparity is the principal topic we wish to address in this paper, with a particular emphasis on what the $R_h = ct$ Universe has to say about the universal expansion implied by the Type Ia supernova data. As we have shown in earlier papers (see, e.g., Melia 2007, 2012a, 2012b, 2012c, 2012d; Melia & Shevchuk 2012), and summarized in §2 below, the $R_h = ct$ Universe is more effective than Λ CDM in accounting for the properties of the CMB. This raises the question of whether Λ CDM is really revealing something different about the local Universe—as opposed to what happened much earlier, closer to the time of last scattering—or whether it is possible for us to understand how and why it may simply be mimicking the expansion history suggested by the $R_h = ct$ Universe at low redshifts.

This is not an easy question to address, principally because of the enormous amount of work that goes into first establishing the supernova magnitudes, and then carefully fitting

the data using the comprehensive set of parameters available to Λ CDM. As we shall see in §3 below, part of the problem is that the distance moduli themselves cannot be determined completely independently of the assumed model. This leads to the unpalatable situation in which the data tend to be somewhat compliant, producing slightly better fits for the input model than might otherwise occur if they were completely model independent. But even with these complications, we will show below that Λ CDM appears to fit the supernova data well because its seven free parameters allow it to relax to the $R_h = ct$ Universe at low redshifts. In other words, we suggest that fits to the Union2.1 sample cannot, by themselves, distinguish between Λ CDM and the $R_h = ct$ Universe. The preponderance of evidence therefore rests with the CMB, which appears to favor the $R_h = ct$ cosmology.

2. The $R_h = ct$ Universe

The $R_h = ct$ Universe is an FRW cosmology that adheres very closely to the restrictions imposed on the theory by the Cosmological Principle and Weyl’s postulate (Melia 2007; Melia & Shevchuk 2012). Most people realize that adopting the Cosmological principle means we assume the Universe is homogeneous and isotropic. But many don’t know that this is only a statement about the structure of the Universe at any given cosmic time t . In order to complete the rationale for the FRW form of the metric, one also needs to know how the time slices at different t relate to each other. In other words, one needs to know whether or not the Cosmological principle is maintained from one era to the next. And this is where Weyl’s postulate becomes essential, for it introduces the equally important assumption about how the various worldlines propagate forwards in time. Weyl’s postulate says that no two worldlines in the Hubble flow ever cross. Small, local crossings are, of course, always possible, and we understand that these are due to small departures from an overall Hubble expansion. But on large scales, the assumption that worldlines never cross

places a severe restriction on how the distance between any two points can change with time. This is the origin of the relationship between the so-called proper distance $R(t)$ in FRW cosmologies and the universal expansion factor $a(t)$, such that $R(t) = a(t)r$, in terms of the unchanging co-moving distance r .

But what has not been recognized until recently is that when these two basic tenets are taken seriously, they force the gravitational horizon R_h (more commonly known as the Hubble radius) to always equal ct . (The proof of this is straightforward and may be found, e.g., in Melia & Shevchuk 2012, and in the more pedagogical treatment of Melia 2012a.) It is not difficult to see that this equality in turn forces the expansion rate to be constant, so the expansion factor $a(t)$ appearing in the Friedmann equations must be t/t_0 (utilizing the convention that $a(t_0) = 1$ today), where t_0 is the current age of the Universe.

Over the past several decades, Λ CDM has developed into a comprehensive description of nature, which nonetheless is not entirely consistent with this approach. Instead, Λ CDM assumes a set of primary constituents (radiation, matter, and an unspecified dark energy), and adopts a partitioning among these components that is not theoretically well motivated. Λ CDM is therefore an empirical cosmology, deriving many of its traits from observations. The problem with this, however, is that the observations are quite varied and cover disparate properties of the Universe, some early in its history (as seen in the fossilized CMB) and others more recently (such as the formation of large-scale structure and the aforementioned distribution of Type Ia supernovae).

In recent work, we have demonstrated why this approach may be useful in establishing a basis for theoretical cosmology, but why in the end Λ CDM essentially remains an approximation to the more precise theory embodied in the $R_h = ct$ Universe. What is lacking in Λ CDM is the overall equation of state, $p = -\rho/3$, linking the total pressure p and the total energy density ρ . This simple relationship is required when the Cosmological

principle and the Weyl postulate are adopted together. In consequence, Λ CDM may “fit” the data in certain restricted redshift ranges, but it fails in other limits. For example, R_h in Λ CDM fluctuates about the mean it would otherwise always have (since it must always be equal to ct), leading to the awkward situation in which the value of $R_h(t_0)$ is equal to ct_0 today, but in order to achieve this “coincidence”, the Universe had to decelerate early on, followed by a more recent acceleration that exactly balanced out the effects of its slowing down at the beginning. In §§ 4 and 5, we shall see how this balancing act functions in a more quantitative manner (particularly when fitting the Type Ia supernova data), and we shall actually learn that the cancellation is even more contrived than it seems at first blush.

The problems with Λ CDM begin right at the outset because of its implied deceleration just after the big bang. This slowing down brings it into direct conflict with the near uniformity of the CMB data (see, e.g., Melia 2012a), requiring the introduction of an ad hoc inflationary phase to rescue it. As shown in Melia (2012b), however, the recent assessment of the observed angular correlation function $C(\theta)$ in the CMB simply does not support any kind of inflationary scenario. We showed in that paper that, whereas Λ CDM fails to account for the observed shape of $C(\theta)$ —particularly the absence of any correlation at angles greater than $\sim 60^\circ$ —the $R_h = ct$ Universe explains where $C(\theta)$ attains its minimum value (at θ_{\min}), the correlation amplitude $C(\theta_{\min})$ at that angle and, most importantly, why there is no angular correlation at large angles. The chief ingredient in Λ CDM responsible for this failure to account for the properties of $C(\theta)$ is inflation, which would have expanded all fluctuations to very large scales. Instead, the $R_h = ct$ Universe did not undergo such an accelerated expansion, so its largest fluctuations are limited to the size of the gravitational horizon $R_h(t_e)$ at the time of last scattering.

To be completely fair to the standard model, however, we also point out that in spite of its failure to account for the distribution of large-scale fluctuations in the CMB, it does

very well in explaining the large- l power spectrum associated with fluctuations on scales below $\sim 1^\circ$. And recent observations provide an even more convincing confirmation of the large- l spectrum predicted by Λ CDM (see, e.g., Hlozek et al. 2012). More work needs to be carried out to fully understand this disparity. It could be that unlike the Sachs-Wolfe effect, the physical processes (such as acoustic fluctuations) influencing the small-scale behavior are much less dependent on the expansion scenario. There is a hint that this may be happening from the analysis of Scott et al. (1995), who demonstrated that the same large- l power spectrum can result from rather different cosmological models.

The CMB carries significant weight in determining which of the candidate models accounts for the universal expansion, and here we have a situation in which Λ CDM cannot explain the near uniformity of the CMB across the sky without inflation, yet with inflation it cannot account for the anisotropy of the CMB on large scales.

This distinction alone already demonstrates the superiority of the $R_h = ct$ Universe over Λ CDM, insofar as the interpretation of the CMB observations is concerned. But this is only one of several crucial tests affirming the conclusion that Λ CDM is only an approximation to the more precise $R_h = ct$ Universe. Here are several other reasons:

- The $R_h = ct$ Universe explains why it makes sense to infer a Planck mass scale in the early Universe by equating the Schwarzschild radius to the Compton wavelength. In Λ CDM there is no justifiable reason why a delimiting gravitational horizon should be invoked in an otherwise infinite Universe (see Melia 2012a for a more pedagogical explanation).

- The $R_h = ct$ Universe explains why $R_h(t_0) = ct_0$ today (because they are always equal). In Λ CDM, this equality is just one of many coincidences. As we shall see in subsequent sections of this paper, this awkwardness will become apparent in how the free parameters in Λ CDM must be manipulated in order to fit the Type Ia supernova data (see, e.g., Figure 4).

- The $R_h = ct$ Universe explains how opposite sides of the CMB could have been in equilibrium at the time ($t_e \sim 10^4 - 10^5$ years) of recombination, without the need to introduce an ad hoc period of inflation. Inflation may be useful for other reasons, but it does not appear to be necessary in order to solve a non-existent “horizon problem” (Melia 2012d).

- The $R_h = ct$ Universe explains why there is no apparent length scale below the BAO wavelength (~ 100 Mpc) in the observed matter correlation function. In their exhaustive study, Watson et al. (2011) concluded that the observed power-law galaxy correlation function below the BAO is simply not consistent with the predictions of Λ CDM, which requires different clustering profiles of matter on different spatial scales. These authors suggested, therefore, that the galaxy correlation function must be a “cosmic coincidence.” But this is not the case in the $R_h = ct$ Universe because this cosmology does not possess a Jeans length (Melia & Shevchuk 2012). Since $p = -\rho/3$, the active gravitational mass ($\rho + 3p$) in the $R_h = ct$ Universe is zero, so fluctuations grow as a result of (negative) pressure only, without any delimiting spatial scale.

- The fact that ρ is partitioned into $\approx 27\%$ matter and $\approx 73\%$ dark energy is a mystery in Λ CDM. But in the $R_h = ct$ Universe, it is clear why Ω_m must be $\approx 27\%$, because when one forces ρ to have the specific constituents ρ_r , ρ_m , and ρ_Λ , only the value $\Omega_m \approx 0.27$ will permit the Universe to evolve in just the right way to satisfy the condition $R_h(t_0) = ct_0$ today. This condition is always satisfied in the $R_h = ct$ Universe, but not in Λ CDM. Yet the observations today must be consistent with this constraint imposed by the Cosmological principle and the Weyl postulate, so all the other evolutionary aspects of the Λ CDM cosmology must comply with this requirement.

- The observed near alignment of the CMB quadrupole and octopole moments is a statistically significant anomaly for Λ CDM, but merely lies within statistically reasonable

expectations in the $R_h = ct$ Universe (Melia 2012c). This again has to do with the finite fluctuation size, limited by the gravitational horizon $R_h(t_e)$ at the time of last scattering.

3. The Union2.1 Supernova Sample

Let us now briefly review the contents of the Union2.1 sample, and summarize the key steps taken during its assembly. As the samples have grown and become better calibrated, evidence has emerged for a correlation between host galaxy properties and SN luminosities, after corrections are made for lightcurve width and SN color (Hicken et al. 2009). For example, Type Ia supernovae in early-type galaxies appear to be brighter (by about 0.14 mag) than their counterparts in galaxies of later type. A similar relationship appears to exist between Hubble residuals and host galaxy mass (Kelly et al. 2010; Sullivan et al. 2010; Lampeitl et al. 2010). Uncorrected, such relationships can lead to significant systematic error in determining the best fit cosmology.

Additional sources of uncertainty arise for astrophysical reasons, including the color correction that must be applied to SN luminosities. The so-called redder-fainter relation apparently arises from at least two mechanisms: extinction from interstellar dust, and probably some intrinsic relation between color and luminosity produced by the explosion itself or by the surrounding medium. It is difficult to justify the argument that this redder-fainter relationship should behave in the same way at all redshifts, but there's little else one can do because the two effects are very difficult to disentangle (see, e.g., Suzuki 2012).

Combining the many available datasets into a single compilation (the Union2.1 sample) has obvious advantages, treating all supernovae on an equal footing and using the same lightcurve fitting, but this process brings its own set of possible errors, including the fact

that the systematics may be different among the various datasets. As we shall see shortly, this multitude of uncertainties makes it impossible to determine the supernova luminosities without some prior assumption about the underlying cosmological model.

The procedure for determining each individual Type Ia supernova luminosity requires a fit to the lightcurve using three parameters (aside from those arising in the cosmological model itself). These are (1) an overall normalization, x_0 , to the time dependent spectral energy distribution of the supernova, (2) the deviation, x_1 , from the average lightcurve shape, and (3) the deviation, c , from the mean Type Ia supernova $B - V$ color. These three parameters, along with the assumed host mass, are then combined to form the distance modulus

$$\mu_B = m_B^{\max} + \alpha \cdot x_1 - \beta \cdot c + \delta \cdot P(m_*^{\text{true}} < m_*^{\text{threshold}}) - M_B, \quad (1)$$

where m_B^{\max} is the integrated B -band flux at maximum light, M_B is the absolute B -band magnitude of a Type Ia supernova with $x_1 = 0$, $c = 0$, and $P(m_*^{\text{true}} < m_*^{\text{threshold}}) = 0$. Also, $m_*^{\text{threshold}} = 10^{10} M_\odot$ is the threshold host-galaxy mass used for the mass-dependent correction, and P is a probability function assigning a probability that the true mass, m_*^{true} , is less than the threshold value, when an actual mass measurement m_*^{obs} is made.

It is quite evident that the task of accurately determining μ_B for each individual supernova is arduous indeed. The Supernova Cosmology Project calls the parameters α , β , δ , and M_B “nuisance” parameters, because they cannot be evaluated independently of the assumed cosmology. They must be fitted simultaneously with the other cosmological parameters, e.g., emerging from Λ CDM.

But this is quite a serious problem that cannot be ignored, e.g., when comparing the overall fits to the data with Λ CDM and the $R_h = ct$ Universe (or any other cosmology, for that matter) because, as we have already alluded to above, this procedure makes the data at least somewhat compliant to the assumed cosmological model.

4. Theoretical Fits

The best fit cosmology is determined by an iterative χ^2 -minimization of the function

$$\chi_{\text{stat}}^2 = \sum_{\text{supernovae}} \frac{[\mu_B(\alpha, \beta, \delta, M_B) - 5 \log 10(d_L(\xi, z)/10 \text{ pc})]^2}{\sigma_{\text{lc}}^2 + \sigma_{\text{ext}}^2 + \sigma_{\text{sample}}^2}, \quad (2)$$

where ξ stands for all the cosmological parameters that define the fitted model (with the exception of the Hubble constant H_0), d_L is the luminosity distance, and σ_{lc} is the propagated error from the covariance matrix of the lightcurve fits. The uncertainties due to host galaxy peculiar velocities, Galactic extinction corrections, and gravitational lensing are included in σ_{ext} (which is evidently of order 0.01 mag), and σ_{sample} is a floating dispersion term containing sample-dependent systematic errors. Its value is obtained by setting the reduced χ^2 to unity for each sample in the compilation. For the Union2.1 catalog, σ_{sample} is approximately 0.15 mag (Suzuki et al. 2012).

The luminosity distance is defined by the relation

$$d_L \equiv a(t_0) r_e \frac{a(t_0)}{a(t_e)}, \quad (3)$$

where $a(t_0)$ is the expansion factor at the present cosmic time t_0 , r_e is the comoving distance to the source, and t_e is the time at which the source at r_e emitted the light we see today.

In Λ CDM, the density is comprised of three primary components,

$$\rho = \rho_r + \rho_m + \rho_\Lambda \quad (4)$$

where, following convention, ρ_r is the density due to radiation, ρ_m is the matter density and ρ_Λ represents dark energy, usually assumed to be a cosmological constant Λ . Dividing through by the critical density today,

$$\rho_c \equiv \frac{3c^2 H_0^2}{8\pi G}, \quad (5)$$

we may write

$$\Omega \equiv \frac{\rho}{\rho_c} = \Omega_r + \Omega_m + \Omega_\Lambda \quad (6)$$

where, in obvious notation, $\Omega_r \equiv \rho_r/\rho_c$, $\Omega_m \equiv \rho_m/\rho_c$, and $\Omega_\Lambda \equiv \rho_\Lambda/\rho_c$.

In a spatially flat universe ($k = 0$), $\Omega = 1$, so if we assume that $\rho_r \sim a^{-4}$, $\rho_m \sim a^{-3}$, and $\rho_\Lambda = \text{constant}$ then, from the geodesic equation,

$$c dt = -a(t) dr , \quad (7)$$

we get

$$r_e = \frac{c}{H_0} \frac{1}{a_0^2} \int_{a_e}^{a_0} \frac{da}{\sqrt{\Omega_r + a\Omega_m/a_0 + (a/a_0)^4\Omega_\Lambda}} , \quad (8)$$

where $a_0 \equiv a(t_0)$ and $a_e \equiv a(t_e)$. And changing the variable of integration to $u \equiv a/a_0$, we find that in ΛCDM

$$d_L^{\Lambda\text{CDM}} = \frac{c}{H_0} (1+z) \int_{\frac{1}{1+z}}^1 \frac{du}{\sqrt{\Omega_r + u\Omega_m + u^4\Omega_\Lambda}} . \quad (9)$$

Figure 1 shows the Union2.1 sample of 580 Type Ia supernovae, along with the best fit ΛCDM model, in which dark energy is a cosmological constant, Λ , its equation-of-state parameter is $w = -1$, and $\Omega_m = 0.27$. As is well known by now, the theoretical fit is excellent. In addition, for these best-fit parameters, the expansion of the Universe switched from deceleration to acceleration at $z \approx 0.75$, which corresponds to a look-back time (in the context of ΛCDM) of ≈ 6.6 Gyr, roughly half the current age of the Universe. Equality between ρ_m and ρ_Λ occurred later, at $z \approx 0.39$, corresponding to a look-back time of about 4.2 Gyr.

However, the purpose of this paper is not so much to demonstrate this well known result but, rather, to see if our suggested resolution of the disparity between the predictions of ΛCDM and the CMB observations is also consistent with the cosmological expansion implied by the Type Ia supernova data. Since the $R_h = ct$ Universe is superior to ΛCDM in at least some fitting of the CMB data, the question now is whether it can also account for the measured expansion at lower redshifts.

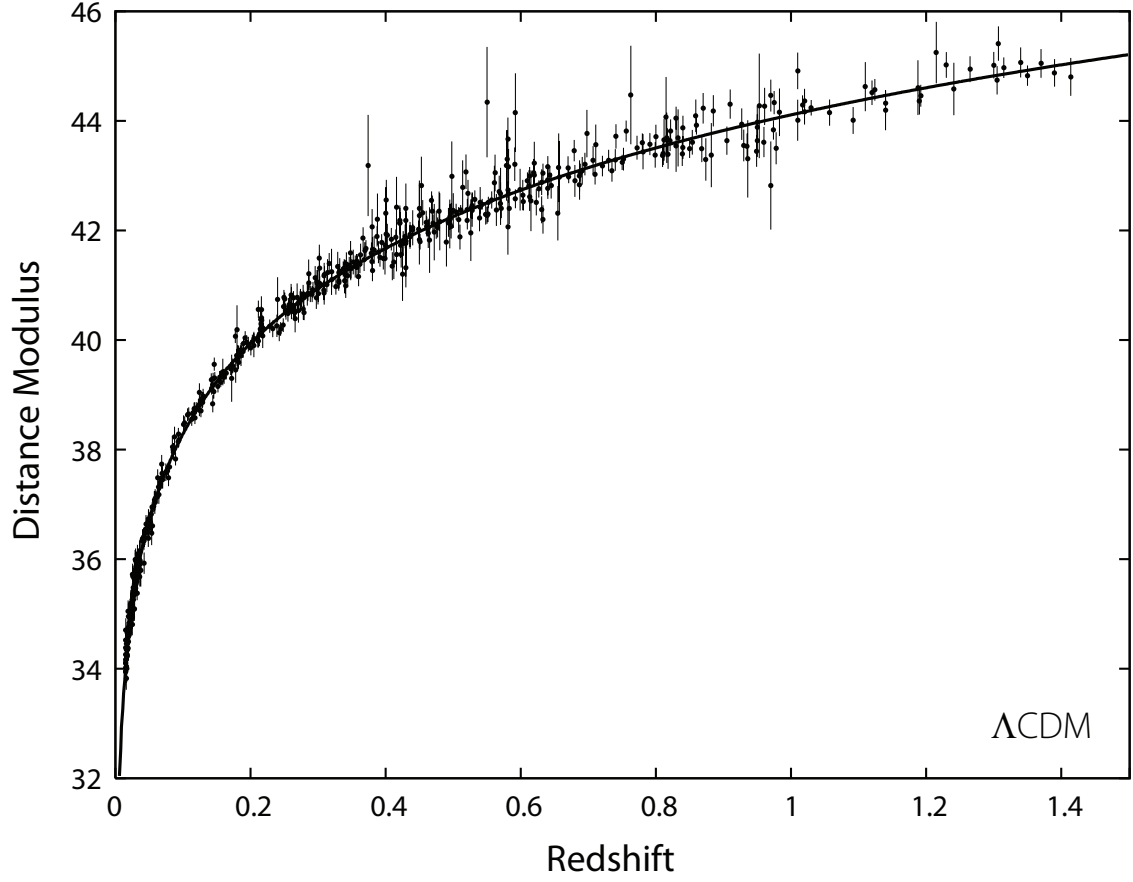


Fig. 1.— Hubble diagram for the Union2.1 compilation (of 580 Type Ia supernova events). The solid curve represents the best-fitted Λ CDM model for a flat universe and matter energy density $\Omega_m = 0.27$ (see text). (Adapted from Suzuki et al. 2012)

For the $R_h = ct$ Universe, we have

$$\frac{a(t_0)}{a(t_e)} = \frac{t_0}{t_e}, \quad (10)$$

from which we get

$$r_e = \ln \left(\frac{t_0}{t_e} \right). \quad (11)$$

And with

$$1 + z = \frac{a(t_0)}{a(t_e)}, \quad (12)$$

it is easy to see that

$$d_L^{R_h=ct} = \frac{c}{H_0} (1 + z) \ln(1 + z) \quad (13)$$

(where we have also used the fact that $t_0 = 1/H_0$).

In carrying out a fit to the data, we note that the chosen value of the Hubble constant H_0 is not independent of M_B . In other words, one can vary either of these parameters, but not both separately. Therefore, if we allow M_B to remain a member of the set of variables optimized to find the “best-fit” supernova luminosities (as described in § 3 above), then the $R_h = ct$ Universe has no free parameters. In contrast, Λ CDM has several, depending on how one chooses to treat dark energy. These include the value of Ω_m and possibly the equation-of-state parameter w , if dark energy is not a pure cosmological constant. In Λ CDM, these parameters affect the location of the Universe’s transition from deceleration to acceleration, and the location where $\rho_m = \rho_\Lambda$.

It is beyond the scope of this paper to carry out a complete best-fit minimization of χ_{stat}^2 for the $R_h = ct$ Universe, which would require an optimization of all the parameters, including α , β , and δ , among others. Instead, we will adopt the parameters identified by the Supernova Cosmology Project as the best-fit values for the Λ CDM model shown in Figure 1, save for one variable—the absolute magnitude M_B . In other words, we will still determine a best fit to the data using the $R_h = ct$ cosmology, but only by minimizing χ_{stat}^2

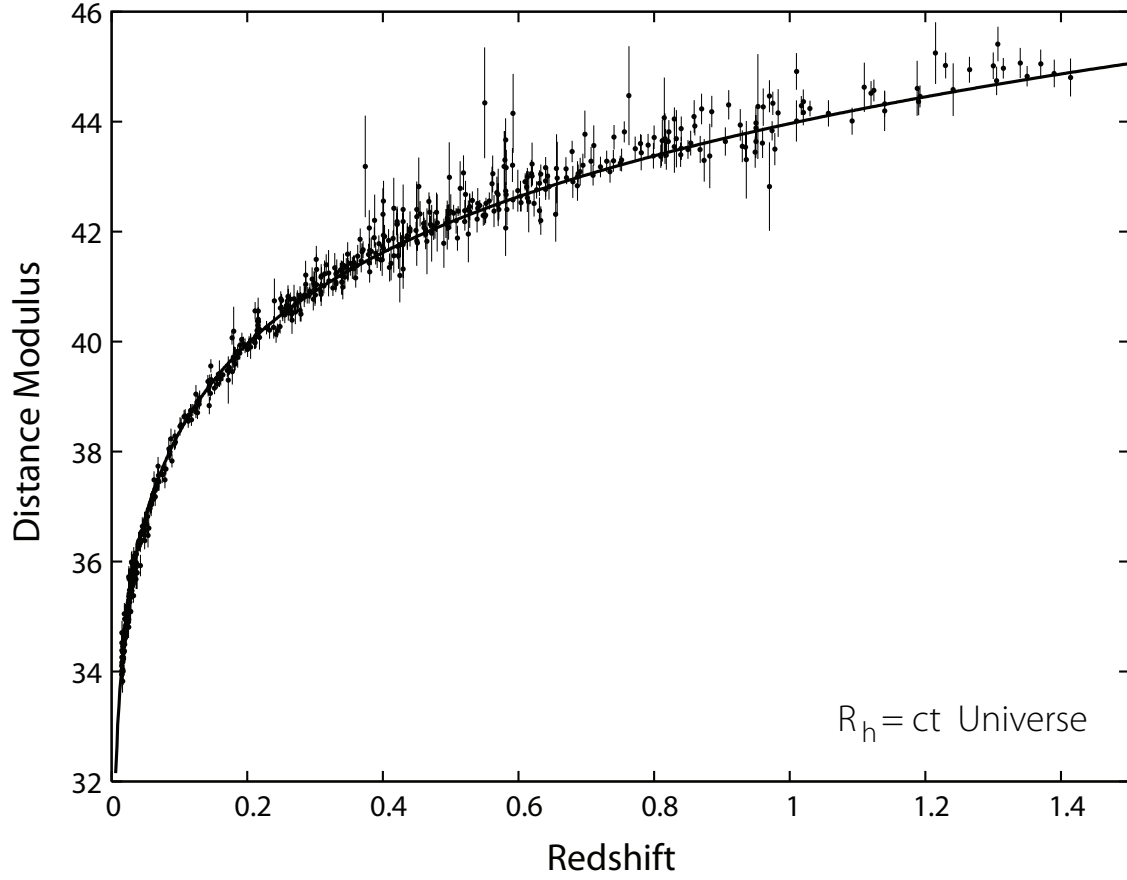


Fig. 2.— Same as Figure 1, except now the solid curve represents the $R_h = ct$ Universe.
(Data are from Suzuki et al. 2012)

with respect to M_B . This is not ideal, of course, because the fit would be even better, were we to allow all of the available parameters to vary, but even this very simplified approach is sufficient to prove our point and it is easy to digest, so it should suffice for now.

The $R_h = ct$ distance profile is compared to the Union2.1 data in Figure 2. The optimization procedure we have just described results in a “best-fit” value for M_B that is 0.126 mag greater than that obtained with Λ CDM. However, for this illustration, all the other parameters are identical to those calculated for Λ CDM (resulting in the fit shown in Figure 1). For completeness, we also show the Hubble diagram residuals corresponding to the $R_h = ct$ cosmology in Figure 3.

5. Discussion

The fits are so similar that even if there weren’t any direct relationship between the two cosmologies, one would have to suspect that there exists yet another “cosmic coincidence” in Λ CDM. But we will now argue that in fact this is not a coincidence—that Λ CDM is merely relaxing to the $R_h = ct$ Universe when all of its parameters are optimized to fit the data. To help with this discussion, let us look at the two distance vs. redshift profiles side by side in Figure 4.

Several characteristics of this comparison are rather striking. First, the two curves are virtually identical, and not only at low redshifts, but all the way out to $z = 6$ or more. They differ slightly around $z \sim 1 - 2$, which should not be surprising given that the two luminosity distances (Equations 9 and 13) depend quite differently on the assumed parameters. Yet, with all the possible deviations one might have expected between these two formulations with increasing z , one instead sees that Λ CDM is forced to track $R_h = ct$ over the entire redshift range.

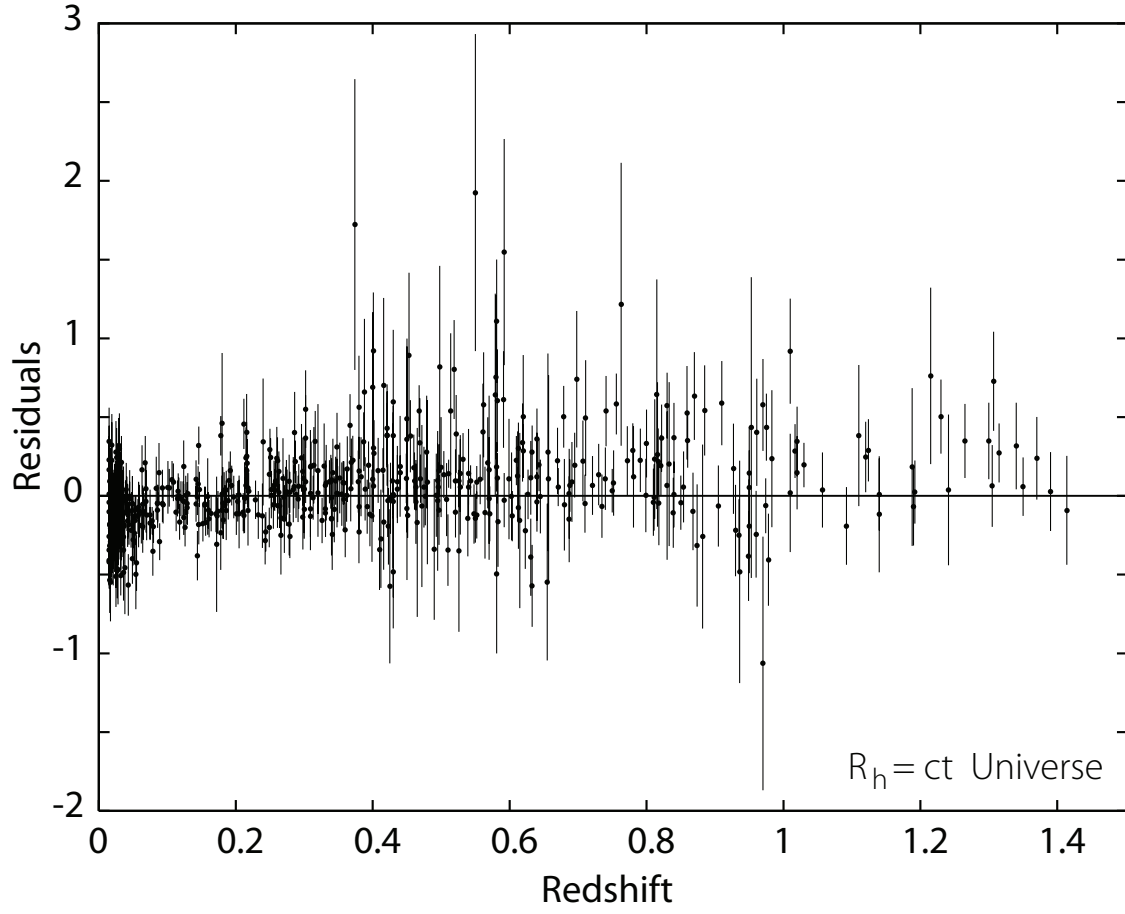


Fig. 3.— Hubble diagram residuals where the $R_h = ct$ Universe profile has been subtracted from the Union2.1 distance moduli. (Data are from Suzuki et al. 2012)

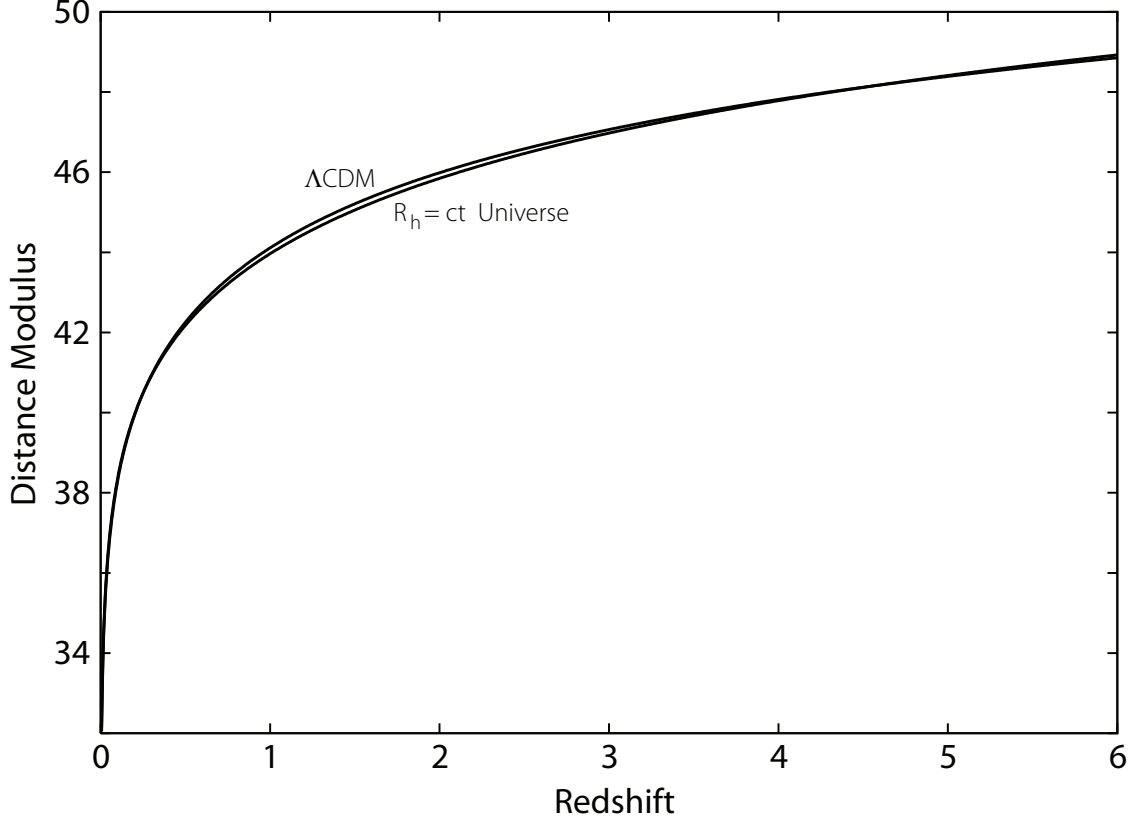


Fig. 4.— A side-by-side comparison of the best fitted Λ CDM and $R_h = ct$ Universe cosmologies shown in Figures 1 and 2, except here extended to a much larger redshift, $z \rightarrow 6$. The most noticeable feature of this comparison is how closely the best-fitted Λ CDM cosmology mimics the $R_h = ct$ Universe, from the present time, t_0 , all the way back to when the Universe was only $t_0/7$ years old. The point is that with all their complexity, the many free parameters in Λ CDM must be adjusted to fit the data in such a way that the universal expansion is essentially what it would have been anyway in the $R_h = ct$ Universe.

Second, a big deal is made of the fact that in Λ CDM, the cosmological expansion switched from deceleration to acceleration at about half its current age. That in itself is a rather strange coincidence. Do we really live at such a special time that we are privileged to have seen the Universe decelerate for half of its existence and then accelerate for the other half, but in such a perfect balance that the two effects exactly canceled out? The reasonable answer to this question is clearly no. And we can see in Figure 4 what must be happening. Since the $R_h = ct$ constraint is imposed on the Universe by the Cosmological principle and the Weyl postulate, even in Λ CDM the cosmological distance scale must comply globally with this condition. So if the imperfect formulation of d_L in Equation 9 first forces a decelerated expansion, a compensating acceleration must take place later in order to reduce the overall cosmological expansion back to $R_h = ct$ as $t \rightarrow t_0$.

But if the $R_h = ct$ curve should be the best fit to the data, why does Λ CDM provide such a good fit to the Union2.1 sample, even at $z \sim 0.7$? We suggest that the problem lies with the so-called “nuisance” variables used to determine the supernova luminosities. When Λ CDM is fit to the data, these variables are optimized along with the other parameters defining the cosmological model (what we called ξ in § 3). With so much flexibility (α , β , δ , and M_B), the distance moduli cannot be determined without the assumption of an underlying cosmology. And if χ_{stat}^2 is minimized by varying all of the parameters simultaneously, it should not be surprising to see the data complying with the model.

A very important contribution to this discussion was made recently by Seikel & Schwarz (2009), who improved the analysis of Type Ia SN data by eliminating some of the uncertainties arising from specific model fitting. Instead of performing a routine χ_{stat}^2 fit using Equation (2), they considered the redshift dependence of the quantity $\Delta\mu_i$ averaged over redshift bins. Each $\Delta\mu_i$ is defined to be the difference between the measured distance modulus of event i and the value it would have had at that redshift if the luminosity distance

were calculated assuming no universal acceleration, i.e., if it were given by Equation (13).

Their goal was simply to ascertain whether or not the average $\langle \Delta\mu \rangle$ is greater than zero—the “null” hypothesis. A positive value would be taken as evidence of recent acceleration. By comparing these quantities at redshifts greater than 0.1 to those below, their goal was to remove as many systematic uncertainties as possible, while at the same time not restricting their analysis to any particular model. The attractive feature of this approach is that one needs only to demonstrate that $\langle \Delta\mu \rangle > 0$ for acceleration to have occurred, without relying on fits with a particular cosmology.

Interestingly, their principal conclusion was that acceleration emerges only if one includes the events at $z < 0.1$. Quite significantly, the evidence for acceleration dramatically (to use their own terminology) decreases if SNe with $z < 0.1$ are not used for the test. In their careful assessment of this effect, they pointed out that a redshift of 0.1 corresponds approximately to a distance of about 400 Mpc, the size of the largest observed structures in the Universe. In other words, the evidence for acceleration emerges only when one includes events associated with a length scale over which the assumption of homogeneity and isotropy is probably not justified.

Having said this, it must be pointed out that although their approach appears to be superior to the simple χ^2_{stat} fitting given in Equation (2), it nonetheless also suffers from the problem we have been describing—the model-dependence of the reduced data. Their $\Delta\mu_i$ are the differences between the measured distance moduli μ_i and the values these would have at the same redshift in a non-accelerated universe. Unfortunately, in order to measure μ_i , one must pre-assume a cosmology. as described in § 3, for otherwise it is not possible to determine the 4 nuisance parameters. And when one adopts a particular cosmology, optimizing its free parameters via the fitting procedure, the nuisance parameters are themselves optimized to comply with that particular cosmology. The values of μ_i used by

Seikel & Schwarz (2009) were therefore not model-independent, as they assumed. Instead, they had already been optimized for Λ CDM, ensuring that they would show at least some vestige of acceleration relative to the null case.

Their work is useful nonetheless because they were apparently able to at least eliminate some of the systematic errors, and in so doing, demonstrate quite compellingly that the elimination of events associated with the inhomogeneous portion of the local Universe greatly decreases the evidence for acceleration.

In any fitting procedure, whether it be a direct χ^2_{stat} fit according to Equation (2), or a more sophisticated approach based on Seikel & Schwarz’s null hypothesis, what is clearly needed is a careful reduction of the data using the very same cosmology being tested. It is not correct to optimize the nuisance parameters using one model and then use them to test the predictions of another. It is our hope that such an exercise will be carried out soon for a direct comparison between Λ CDM and the $R_h = ct$ Universe.

6. Conclusions

The analysis of Type Ia supernova data over the past decade has been one of the most notable success stories in cosmology. These are arguably the most reliable standard candles we have to date and offer us an unparalleled opportunity of studying the cosmological expansion, at least out to a redshift of ~ 1.5 . The consensus today appears to be that Λ CDM offers the best explanation for the luminosity-distance relationship seen in these events, and many believe that what remains to be done is simply to refine the best-fit values of the parameters that define this model.

One may therefore question the value of re-examining the merit of Λ CDM in these studies, and it would be difficult to justify re-testing the basic theory, were it not for

the significant incompatibility emerging between the standard model and other equally important observations, such as the CMB anisotropies mapped by the Wilkinson Microwave Anisotropy Probe (Bennett et al. 2003). In this paper, we have documented several compelling reasons for believing that Λ CDM does not provide an accurate representation of the cosmological expansion, at least not at high redshift ($z \gg 2$). It has therefore been essential to re-analyze the Type Ia supernova data in light of the cosmology (the $R_h = ct$ Universe) that best represents the Universe’s dynamical evolution at early times.

By directly comparing the distance-relationship in Λ CDM with that predicted by the $R_h = ct$ Universe, and each with the Union2.1 sample, we have demonstrated that the two theories produce virtually indistinguishable profiles. If these were not related, such a close empirical kinship would suggest the emergence of yet another “cosmic coincidence” in Λ CDM. Instead, the fact that the best fit parameters compel Λ CDM to track $R_h = ct$ so closely suggests that the former cosmology is an approximation to the latter, and relaxes to it as best as the formulation in terms of Ω_m , Ω_r , Ω_Λ , and w allows it to.

But because the relaxation is not perfect, Λ CDM accelerates in certain regions and decelerates in others, though always precisely balanced in order to guarantee a net zero acceleration over the age of the Universe. Were it not for the constraint imposed on the gravitational radius by the Cosmological principle and the Weyl postulate, this balancing act would be one more cosmic coincidence, as would the fact that the switch from deceleration to acceleration occurs at the mid-point of the Universe’s existence, endowing us with the remarkable privilege of viewing this transition at a very special time—one that occurs once, and once only, in the entire history of the Universe.

Through this exercise, we have also highlighted the fact that the data themselves cannot be determined independently of the assumed cosmological model. The fact that the supernova luminosities must be evaluated by optimizing 4 parameters simultaneously with

those in the adopted model means that the resultant data are compliant to the implied cosmology. Though this approach is highly unpalatable, there doesn't seem to be any reasonable alternative at this time. However, the dependence of the data on the assumed model suggests that one should not ignore the model-dependent data reduction in any comparative studies between competing theories.

It would therefore be highly desirable to carry out a full fitting analysis of the Union2.1 sample using the $R_h = ct$ Universe, on par with the efforts already undertaken for Λ CDM. Only then can we see whether the Type Ia supernova data reveal a cosmological expansion at low redshifts consistent with the dynamics implied by the CMB anisotropies, or whether they do in fact demonstrate a different behavior at early and late times in the Universe's history.

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